Recovering a Holographic Geometry from Entanglement

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Quantum Gravity from AdS/CFT

An ambitious question

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- Hard to even begin to answer because we don't know what the full formulation of such a theory is!
- We need a framework in which to work: in context of string theory, AdS/CFT gives us a nonperturbative, indirect *definition* of a theory of quantum gravity

Quantum Gravity from AdS/CFT

AdS/CFT Correspondence [Maldacena]

A nonperturbative, background-independent theory of quantum gravity with asymptotically (locally) anti-de Sitter boundary conditions – the "bulk" – is dual to a conformal field theory – the "boundary" – living on (a representative of the conformal structure of) the asymptotic boundary of the bulk.

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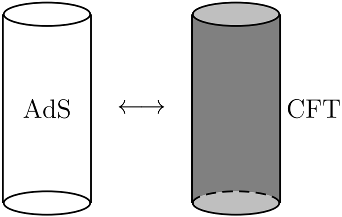
Work around a limit in which the bulk is well-approximated by a classical geometry:



The Holographic Dictionary

Using AdS/CFT as a framework, we can refine the question:

A slightly less vague question

In AdS/CFT, when and how does (semi)classical gravity emerge from the boundary field theory?

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In AdS/CFT, when and how does (semi)classical gravity emerge from the boundary field theory?

- Requires understanding what "dual" means: the holographic dictionary
- Going from the bulk to the boundary is pretty well-understood (e.g. one-point functions of local boundary operators are given by the asymptotic behavior of local bulk fields)
- Going from the boundary to the bulk is harder: this is broadly termed "bulk reconstruction"

A Line of Attack

The (semi)classical gravity we observe in our universe emerges from some more fundamental quantum theory - how?

$\Downarrow$ (AdS/CFT)

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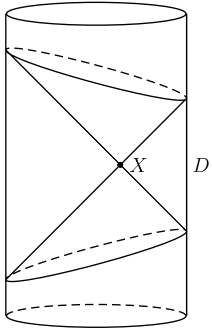
Reconstruction of Bulk Operators

- In pure AdS, local field operators can be expressed in terms of local boundary operators by integrating against a kernel [Hamilton, Kabat, Lifschytz, Lowe]:

$$\phi(X) = \int_{D\subset\partial M} d^{d-1}x\, K(X|x)\mathcal{O}(x)$$



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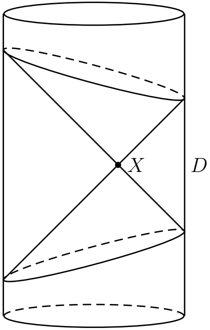

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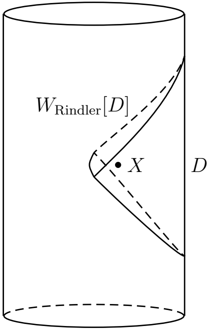

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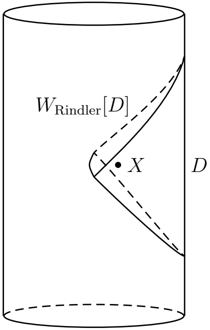

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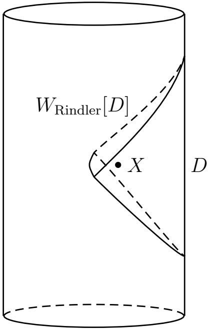
- Stronger hint comes from entanglement entropy



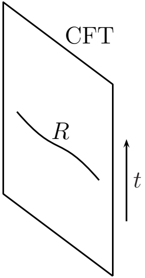
Holographic Entanglement Entropy

HRT Formula [Ryu, Takayanagi, Hubeny, Rangamani]

If $\rho_R = \mathrm{Tr}_{\overline{R}}\,\rho$ is the reduced state associated to some region R and the bulk is well-approximated by a classical geometry obeying Einstein gravity, then

$$S[R] \equiv -\mathrm{Tr}(\rho_R \ln \rho_R) = \frac{\mathrm{Area}[X_R]}{4G\hbar},$$

where X_R is the smallest-area codimension-two extremal surface anchored to ∂R.



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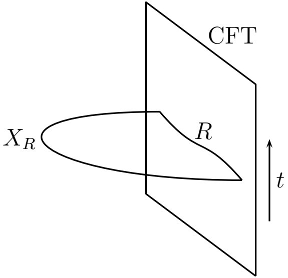

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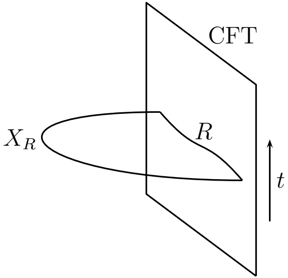
- X_R is generically spacelike separated from the causal diamond $D[R]$, so R is sensitive to more of the bulk than expected from just causal structure



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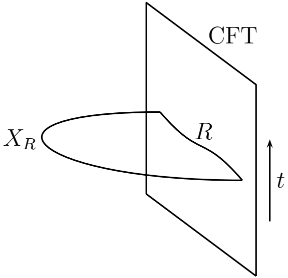
- Ideas from quantum error correction show that X_R defines the region of the bulk to which R is sensitive: bulk operators in the *entanglement wedge* defined by X_R can be represented by CFT operators in $D[R]$

  [Dong, Harlow, Wall; Faulkner, Lewkowycz]



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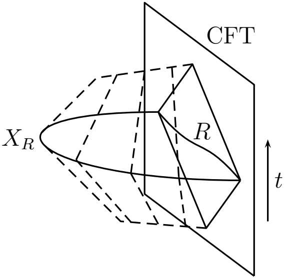

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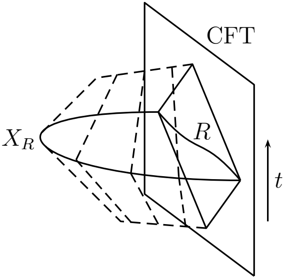
- What about recovering the bulk geometry itself and its properties?



Moving Up

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Recovering the Geometry

The HRT formula clearly connects bulk geometry to boundary entanglement, and its key role in recovering bulk operators on a fixed background strongly suggests it should play a role in recovering the geometry as well [Van Raamsdonk]. Does it?

Recovering the Geometry

Some partial progress:

- Dynamics: For perturbations of vacuum, HRT implies the perturbative Einstein equations in the bulk [Lashkari, Faulkner, Guica, Hartman, McDermott, Myers, Van Raamsdonk, ...]

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- Gravitational thermodynamics: generic area laws in the bulk can be related to monotonicity properties of entropy
  - A coarse-grained entropy defined by fixing a portion of the bulk geometry gives area law along (spacelike) foliations of apparent horizons [Engelhardt, Wall]
  - Casini-Huerta c-theorem relates to mixed-signature area laws in bulk, including along early-time event horizons of black holes formed from collapse [Engelhardt, SF]

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- Causal structure: instead of EE, can use the singularity structure of boundary correlators to deduce the causal structure of (part of) the causal wedge of the bulk [Engelhardt, Horowitz; Engelhardt, SF]

Recovering the Geometry

Here, I'm interested in a more fine-grained question:

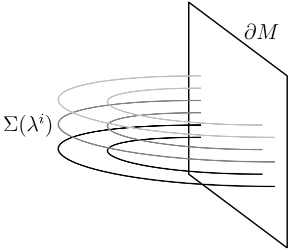
Does knowledge of the entanglement entropy of all regions (i.e. the areas of all HRT surfaces) determine the bulk geometry? How?

- Obviously EE can't recover the full geometry, since there can be regions that HRT surfaces don't reach
- The general expectation has been that EE can recover geometry wherever HRT surfaces reach, but never understood in detail

A Geometric Problem

Assumptions:

- Dimension of bulk geometry M is $d \geq 4$, with *finite* boundary ∂M
- A portion $\mathcal{R}$ of M is foliated by a continuous $(d-2)$-parameter family $\{\Sigma(\lambda^i)\}$ of (planar) two-dimensional spacelike extremal surfaces anchored to ∂M



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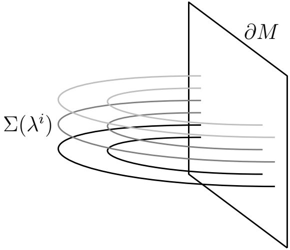

Claim: the geometry in $\mathcal{R}$ is uniquely fixed by the metric and extrinsic curvature of ∂M, the curves $\partial\Sigma(\lambda^i)$, and the variations of the areas of the $\Sigma(\lambda^i)$

Overview of Argument

Four steps, inspired by [Alexakis, Balehowsky, Nachman], using inverse boundary value problems (same sort of techniques used in e.g. medical imaging or geophysics)

1. Gauge fix: introduce a unique coordinate system $\{\lambda^i, x^\alpha\}$ in the region $\mathcal{R}$, with the x^α conformally flat coordinates on $\Sigma(\lambda^i)$: $ds^2_\Sigma = e^{2\phi}[(dx^1)^2 + (dx^2)^2]$

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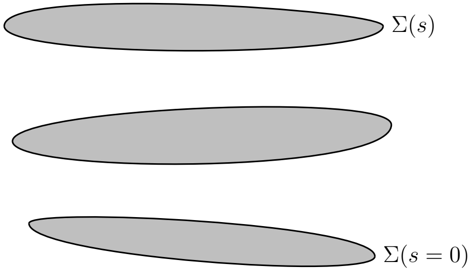
4. The requirement that the $\Sigma(\lambda^i)$ all be extremal yields a hyperbolic evolution equation for ϕ, which has a unique solution

The Jacobi (or Stability) Operator



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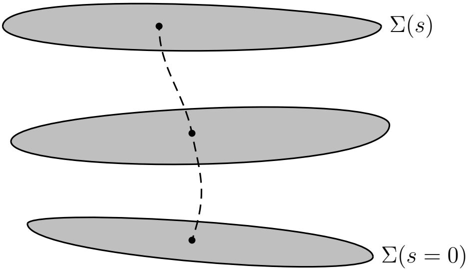

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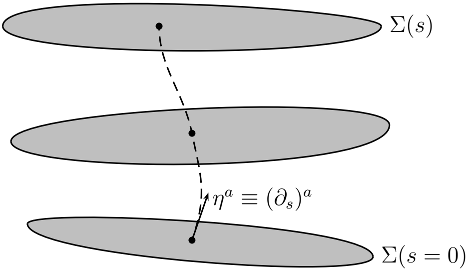

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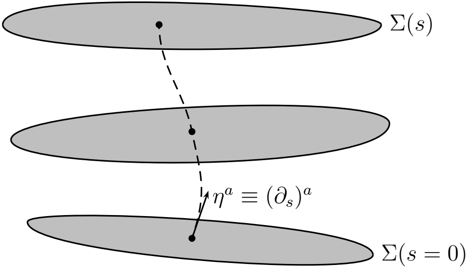

If $\Sigma(s)$ are all geodesics with tangent t^a, deviation vector η^a obeys the equation of geodesic deviation

$$0 = t^b \nabla_b (t^c \nabla_c \eta^a) + R_{bcd}{}^a t^b t^d \eta^c$$

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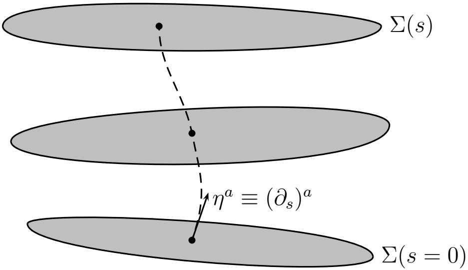

If $\Sigma(s)$ are all extremal surfaces, deviation vector η^a obeys the Jacobi equation

$$0 = D^2\eta^a + \underbrace{\left(K^{acd}K_{bcd} + P^{ac}\sigma^{de}R_{cdbe}\right)}_{\text{curvature terms } \equiv Q^a{}_b}\eta^b \equiv J\eta^a$$

The Jacobi (or Stability) Operator

- Second variations of the area of an extremal surface Σ under deformations of $\partial\Sigma$ give information about its Jacobi operator
- Extend Σ to an arbitrary two-parameter family $\Sigma(s_1, s_2)$ of extremal surfaces, with $\Sigma(0,0) = \Sigma$ and deviation vectors $\eta_1^a = (\partial_{s_1})^a$, $\eta_2^a = (\partial_{s_2})^a$

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- The variation of the area $A(s_1, s_2)$ is a boundary term:

$$\left.\frac{\partial^2 A}{\partial s_1 \partial s_2}\right|_{s_1=0=s_2} = \int_{\partial\Sigma} \eta_2^a D_N(\eta_1)_a + (\text{known boundary stuff})$$

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- So knowing how the area varies as the shape of $\partial\Sigma$ is varied yields the Dirichlet-to-Neumann map of J:

$$\Psi : \eta^a|_{\partial\Sigma} \mapsto D_N \eta^a|_{\partial\Sigma} \text{ such that } J\eta^a = 0$$

Elliptic Inverse Boundary Value Problems fix g^{ij}

- Inverse boundary value problem: if $D_1^\dagger D_1 + Q_1$ and $D_2^\dagger D_2 + Q_2$ acting on a vector bundle on a Riemann surface have the same Dirichlet-to-Neumann map, then D_1, D_2 and Q_1, Q_2 are the same (up to gauge) [Albin, Guillarmou, Tzou, Uhlmann]

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- Use this to fix the basis $\{(n^i)_a\}$ on the normal bundle by requiring that $(n^i)_a(\partial_{\lambda^j})^a = \delta^i{}_j$, which fixes $(n^i)_a = (d\lambda^i)_a$

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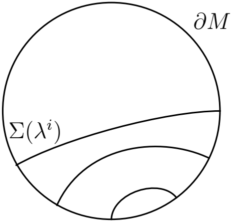
- Metric-compatibility of connection in this gauge requires $D_a g^{ij} = 0$, which fixes g^{ij}

Tilting Fixes $g^{\alpha i}$

- Intuition: since the $g^{\alpha i}$ know about "mixing" between directions normal and tangent to $\Sigma(\lambda^i)$, we can mix them by "tilting" the folitation $\Sigma(\lambda^i)$ to a family of foliations $\Sigma(s; \lambda^i_s)$:



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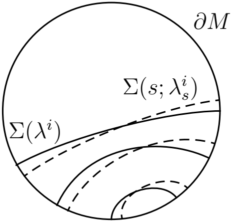

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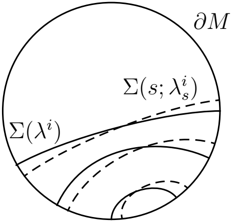
- Intuition: since the $g^{\alpha i}$ know about "mixing" between directions normal and tangent to $\Sigma(\lambda^i)$, we can mix them by "tilting" the folitation $\Sigma(\lambda^i)$ to a family of foliations $\Sigma(s;\lambda^i_s)$:



- The $\{\lambda^i_s, x^\alpha_s\}$ give a new coordinate system (related to the $\{\lambda^i, x^\alpha\}$ by a diffeomorphism generated by η^a):

$$\lambda^i_s(p) = \lambda^i(p) + s\eta^i(p) + \mathcal{O}(s^2), \qquad x^\alpha_s(p) = x^\alpha(p) + s\dot{x}^\alpha(p) + \mathcal{O}(s^2)$$

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- Expanding $g_s^{ij} \equiv g^{ab}(d\lambda_s^i)_a(d\lambda_s^j)_b$ to first order in s, get

$$\underbrace{\left.\frac{dg_s^{ij}}{ds}\right|_{s=0} - 2g^{k(i}\partial_k\eta^{j)} - \eta^k\partial_k g^{ij}}_{\text{known}} = \underbrace{\dot{x}^\alpha\partial_\alpha g^{ij} + 2g^{\alpha(i}\partial_\alpha\eta^{j)}}_{\text{linear in unknowns } \dot{x}^\alpha,\ g^{\alpha i}}$$

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- For $d \geq 4$, there are enough linear equations to determine all the unknowns, and can recover $g^{\alpha i}$ ($d = 3$ case studied by [Alexakis, Balehowsky, Nachman] is much harder – need to compute the deformation $\dot{x}^\alpha$ of the isothermal coordinates)

A Hyperbolic PDE Fixes ϕ

- Since $g_{\alpha\beta} = e^{2\phi}\delta_{\alpha\beta}$, the extremality condition requires

$$K^i = 0 \quad \Rightarrow \quad \partial_\alpha f^\alpha{}_i + 2 f^\alpha{}_i \partial_\alpha \phi - 2\partial_i \phi = 0 \qquad (*)$$

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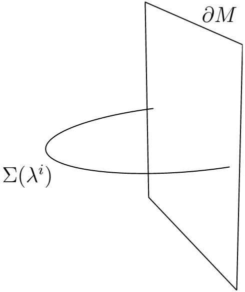

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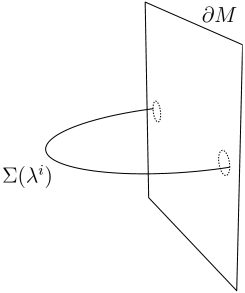

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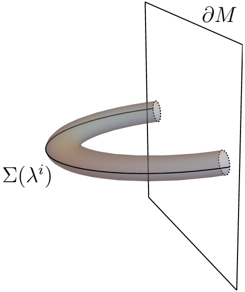

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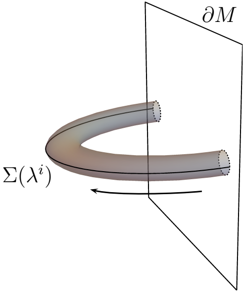
- Construct some periodic cycle in the λ^i, corresponding to a “tube” swept out by the $\Sigma(\lambda^i)$
- Evolve $(*)$ inwards from the boundary along this tube to fix ϕ uniquely on every $\Sigma(\lambda^i)$ on it



Features

- Applies to two-dimensional extremal surfaces in an ambient spacetime of any dimension (and signature), but relevance to bulk reconstruction is in $d = 4$, since then HRT surfaces are two-dimensional

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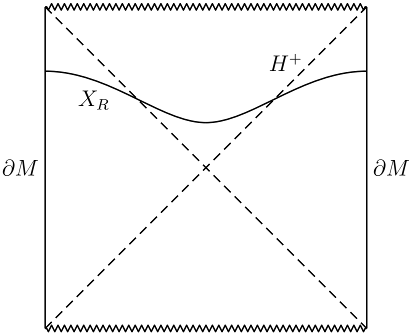
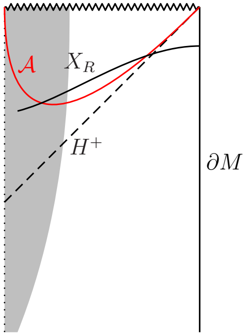
  - Gives (highly implicit) *necessary* conditions for the existence of a dual geometry

- Only requires knowledge of *variations* of entropy, not the actual entanglement entropy of any region

Features

- Can probe inside black holes!





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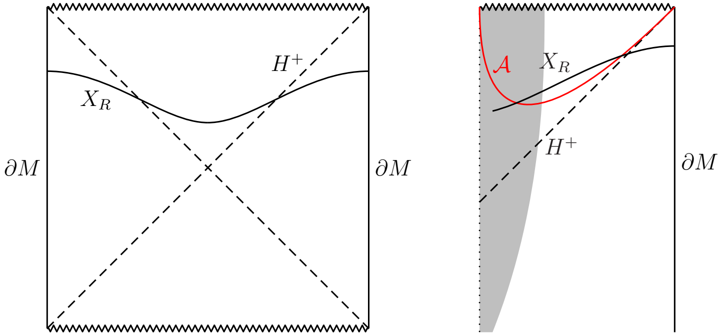

- But still stays away from singularities...

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- Almost: a closely-related result gives an *explicit* method for recovering Q from the Dirichlet-to-Neumann map of the operator $\nabla^2 + Q$ on some domain in $\mathbb{R}^2$, where ∇^2 is the usual (flat-space) Laplacian [Novikov, Santacesaria]
- Generalizing this to our case would give an explicit algorithm for recovering the metric from boundary entanglement

Moving Further Up

The (semi)classical gravity we observe in our universe emerges from some more fundamental quantum theory - how?

$\Downarrow$ (AdS/CFT)

In AdS/CFT, how do the CFT degrees of freedom rearrange themselves to look like a gravitational theory?

$\Downarrow$ **(classical limit)**

When and how does (semi)classical gravity emerge from the boundary field theory?

$\Downarrow$ (probe limit)

How are operators on a *fixed* bulk geometry recovered?

Quantum Corrections to EE

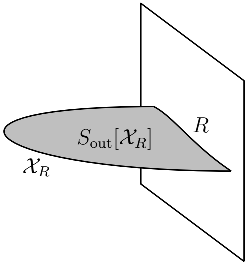
Sub-leading effects in $G\hbar$ ($1/N^2$ in CFT) introduce corrections:

Engelhardt-Wall Formula

Under perturbative quantum corrections,

$$S[R] = S_{\text{gen}}[\mathcal{X}_R] \equiv \frac{\text{Area}[\mathcal{X}_R]}{4G\hbar} + S_{\text{out}}[\mathcal{X}_R],$$

where $\mathcal{X}_R$ is anchored to ∂R and extremizes S_{gen} (a "quantum extremal surface"), and $S_{\text{out}}[\mathcal{X}_R]$ is the von Neumann entropy of any bulk quantum fields "outside" $\mathcal{X}_R$ [Faulkner, Lewkowycz, Maldacena; Dong, Lewkowycz]



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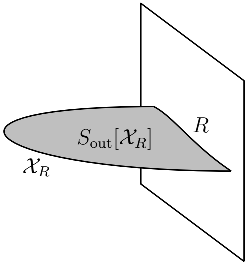

(Note: $\mathcal{X}_R$ can reach further into the bulk than X_R, e.g. late-time horizons of evaporating black holes [Almheiri, Engelhardt, Marolf, Maxfield; Penington])

Incorporating Quantum Effects

- In order to really start probing quantum gravity effects, should be keeping track of these corrections!
- Can consider the same sort of setup, but with foliation by classical extremal surfaces X_R replaced by quantum extremal surfaces $\mathcal{X}_R$

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$$D^2\eta^a + Q^a{}_b\eta^b + 4G\hbar \int_{\Sigma'} P^{ab} \frac{\mathcal{D}^2 S_{\text{out}}}{\mathcal{D}\Sigma^c(p')\mathcal{D}\Sigma^b} \eta^c(p') = 0,$$

with $\mathcal{D}S_{\text{out}}/\mathcal{D}\Sigma^a$ a functional derivative

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- Are D and $\widetilde{Q}^a{}_b$ determined by boundary data just as they are in the classical case? If so, can generalize argument

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- Perturbations give rise to a generalized Jacobi operator $\widetilde{J}$ that depends on the perturbed area functional
- If $\widetilde{J}$ be recovered from boundary data, can likewise generalize the argument to recover the bulk even when it includes these higher-curvature corrections

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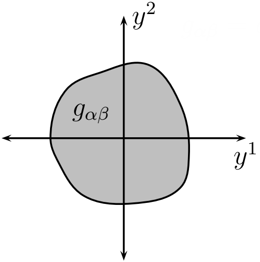
- Even geometry where HRT surfaces don't reach should be recoverable somehow; from what? Quantum extremal surfaces? Other measures of entanglement?
- More generalizations: $(d-2)$-dimensional surfaces in higher dimension d; higher-curvature corrections; how generic is the assumption of a foliation?

Fixing Conformally Flat Coordinates



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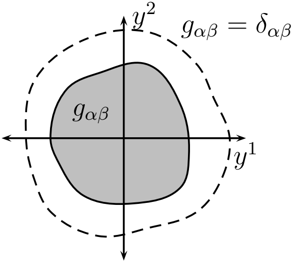

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- Extend $g_{\alpha\beta}$ to all of $\mathbb{R}^2$ so that $g_{\alpha\beta} = \delta_{\alpha\beta}$ away from Σ, and $g_{\alpha\beta}$ is known everywhere outside Σ
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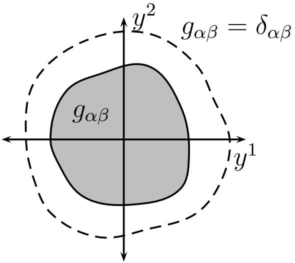

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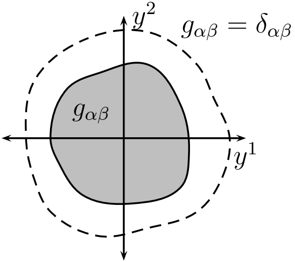

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So for any two metrics g_1, g_2 on Σ with the same boundary data, there exists a set of coordinates $\{x^\alpha\}$ on Σ in which both are conformally flat:

$$ds_\Sigma^2 = e^{2\phi}\left((dx^1)^2 + (dx^2)^2\right)$$