On-Shell Methods for Scattering Amplitudes: Examples

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with support from the Ambrose Monell Foundation
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Kodosky Lectures, Austin, Texas
February 2–4, 2016

Renormalization Scale

- Needed to define the coupling
- Physical quantities should be independent of it
- Truncated perturbation theory isn’t
- Dependence is ~ the first missing order * logs
- Similarly for factorization scale — define parton distributions

Every sensible observable has an expansion in αs

$$\frac{d\sigma}{d\mathcal{O}} = \alpha_s^{n_0}(\mu)\frac{d\hat{\sigma}^{\mathrm{LO}}}{d\mathcal{O}} + \alpha_s^{n_0+1}(\mu)\frac{d\hat{\sigma}^{\mathrm{NLO}}(\mu)}{d\mathcal{O}} + \alpha_s^{n_0+2}(\mu)\frac{d\hat{\sigma}^{\mathrm{NNLO}}(\mu)}{d\mathcal{O}}$$

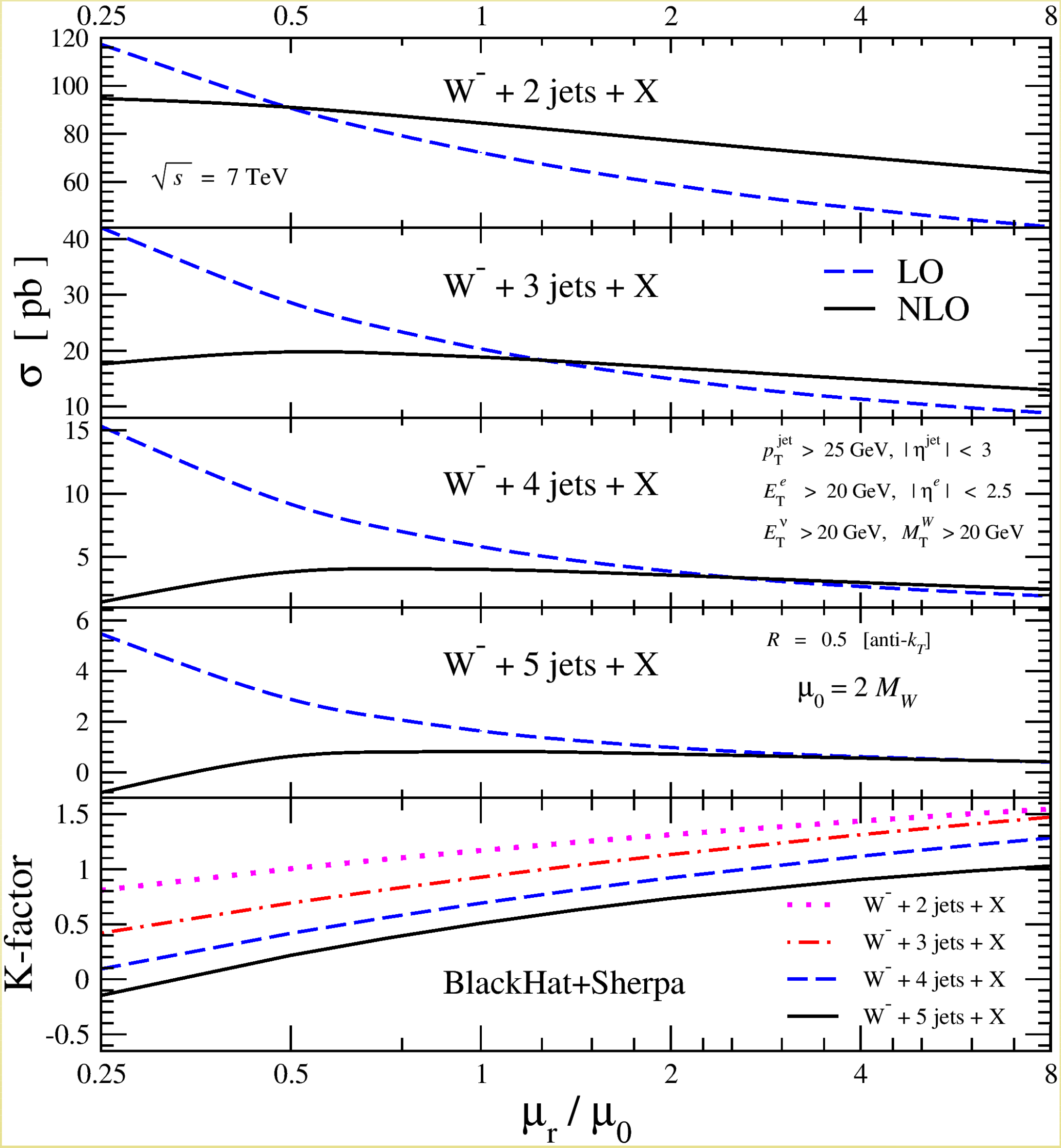
Examples

$$\frac{d\sigma^{W+1\,\mathrm{jet}}}{dp_{\mathrm{T}}^{\mathrm{jet}}} = \alpha_s(\mu)\frac{d\hat{\sigma}^{\mathrm{LO}}}{dp_{\mathrm{T}}^{\mathrm{jet}}} + \alpha_s^2(\mu)\frac{d\hat{\sigma}^{\mathrm{NLO}}(\mu)}{dp_{\mathrm{T}}^{\mathrm{jet}}} + \alpha_s^3(\mu)\frac{d\hat{\sigma}^{\mathrm{NNLO}}(\mu)}{dp_{\mathrm{T}}^{\mathrm{jet}}}$$

$$\frac{d\sigma^{W+2\,\mathrm{jet}}}{dp_{\mathrm{T}}^{\mathrm{2^{nd}\,jet}}} = \alpha_s^2(\mu)\frac{d\hat{\sigma}^{\mathrm{LO}}}{dp_{\mathrm{T}}^{\mathrm{2^{nd}\,jet}}} + \alpha_s^3(\mu)\frac{d\hat{\sigma}^{\mathrm{NLO}}(\mu)}{dp_{\mathrm{T}}^{\mathrm{2^{nd}\,jet}}} + \alpha_s^4(\mu)\frac{d\hat{\sigma}^{\mathrm{NNLO}}(\mu)}{dp_{\mathrm{T}}^{\mathrm{2^{nd}\,jet}}}$$

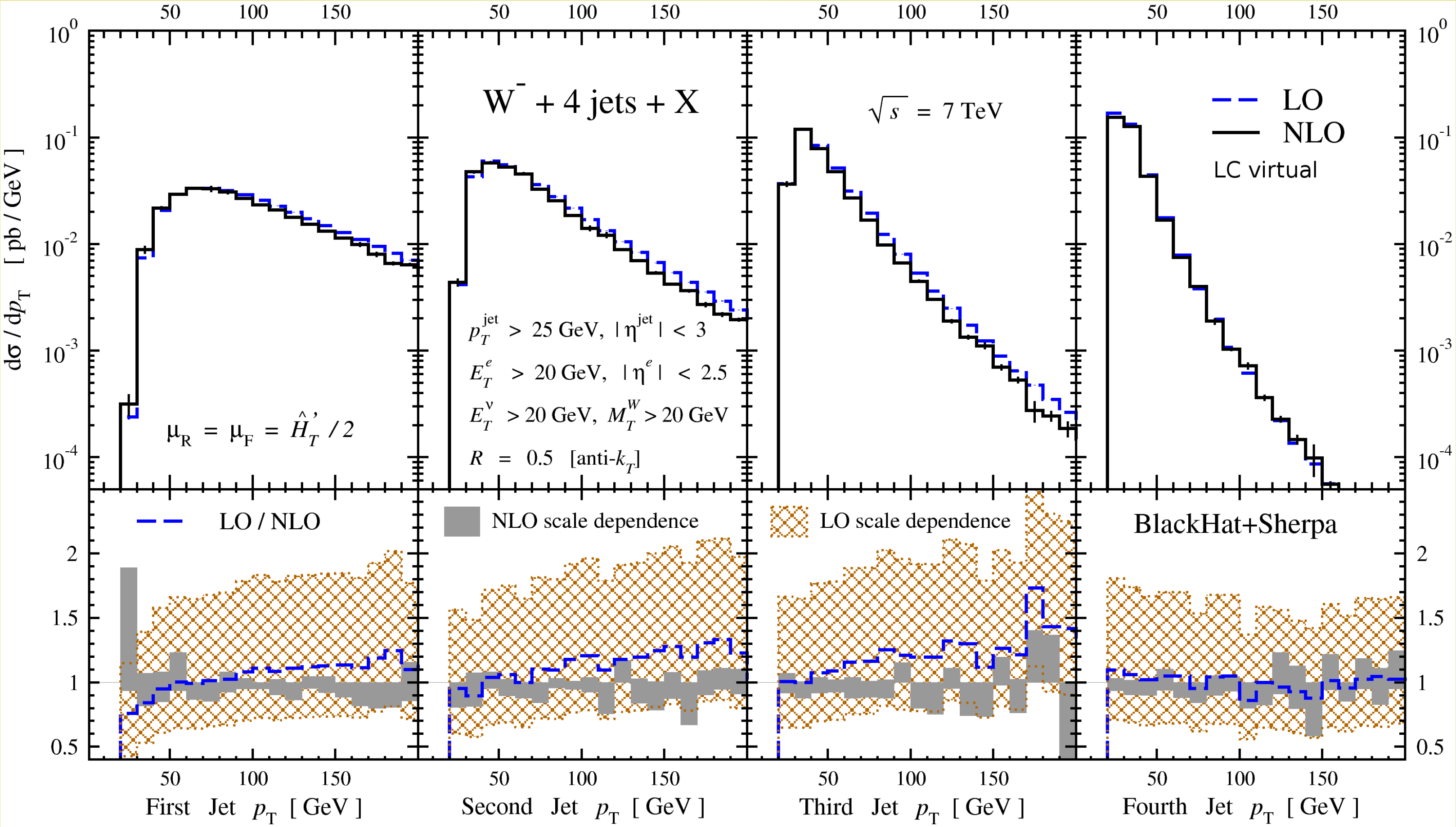
Leading-Order, Next-to-Leading Order

- QCD at LO is not quantitative
- LO: Basic shapes of distributions
  but: no quantitative prediction — large dependence on unphysical
  renormalization and factorization scales
  missing sensitivity to jet structure & energy flow
- NLO: First quantitative prediction, expect it to be reliable to 10–15%
  improved scale dependence — inclusion of virtual corrections
  basic approximation to jet structure — jet = 2 partons
  importance grows with increasing number of jets
- NNLO: Precision predictions
  small scale dependence
  better correspondence to experimental jet algorithms
  understanding of theoretical uncertainties
  will be required for <5% predictions for future precision measurements



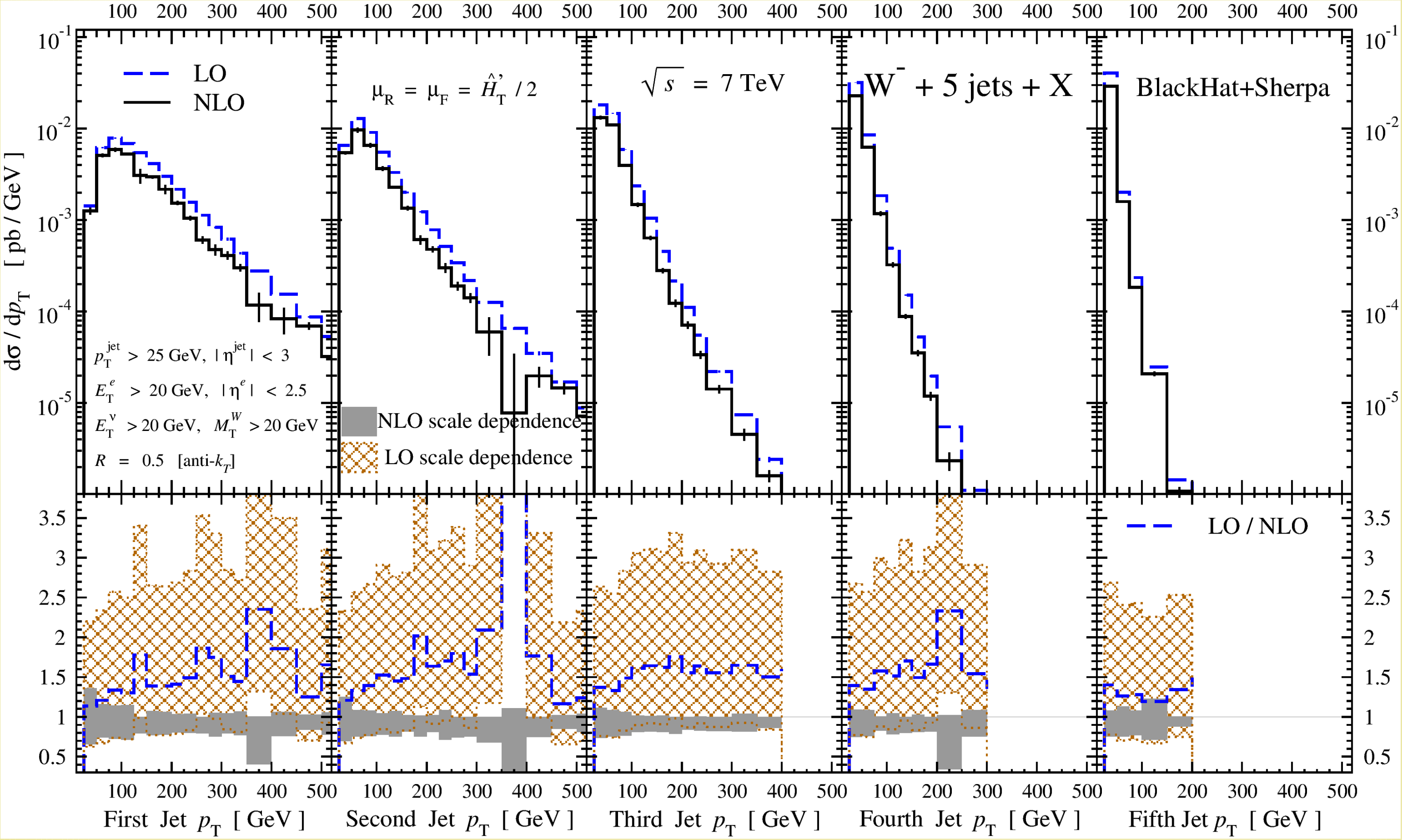
W⁻ + 2 jets + X
√s = 7 TeV
W⁻ + 3 jets + X
LO
NLO
σ [pb]
W⁻ + 4 jets + X
p_T^{jet} > 25 GeV, $|\eta^{jet}|$ < 3
E_T^e > 20 GeV, $|\eta^e|$ < 2.5
E_T^ν > 20 GeV, M_T^W > 20 GeV
W⁻ + 5 jets + X
R = 0.5 [anti-k_T]
$\mu_0 = 2\,M_W$
K-factor
BlackHat+Sherpa
W⁻ + 2 jets + X
W⁻ + 3 jets + X
W⁻ + 4 jets + X
W⁻ + 5 jets + X
μ_r / μ_0

W+4 Jets



- Scale variation reduced substantially at NLO
- Successive jet distributions fall more steeply
- Shapes of 4th jet distribution unchanged at NLO — but first three are slightly steeper

W+5 Jets



LO
NLO
$\mu_R = \mu_F = \hat{H}_T' / 2$
$\sqrt{s} = 7$ TeV
W^- + 5 jets + X
BlackHat+Sherpa
$d\sigma / dp_T$ [pb / GeV]
$p_T^{jet} > 25$ GeV, $|\eta^{jet}| < 3$
$E_T^e > 20$ GeV, $|\eta^e| < 2.5$
$E_T^\nu > 20$ GeV, $M_T^W > 20$ GeV
$R = 0.5$ [anti-k_T]
NLO scale dependence
LO scale dependence
LO / NLO
First Jet p_T [GeV]
Second Jet p_T [GeV]
Third Jet p_T [GeV]
Fourth Jet p_T [GeV]
Fifth Jet p_T [GeV]

Extrapolating